MATH REVIEW BOOKLET

Math And Science Department

Contents

Topic 1- Basic Arithmetic

Vocabulary: **sum, difference, product, quotient, remainder**

Addition:

1) $$\begin{array}{r} 736 \\ +\,123 \\ \hline 859 \end{array}$$

2) $624 + 111 + 231 \longrightarrow$ $$\begin{array}{r} 624 \\ 111 \\ +\ \ 231 \\ \hline 966 \end{array}$$

3) $$\begin{array}{r} {\scriptstyle 1\ 1}\ \ \\ 298 \\ +\ 457 \\ \hline 755 \end{array}$$

4) $1493 + 24 + 787 \longrightarrow$ $$\begin{array}{r} {\scriptstyle 1\ 2\ 1}\ \ \\ 1493 \\ 24 \\ +\ \ 787 \\ \hline 2304 \end{array}$$

Subtraction:

5) $$\begin{array}{r} 878 \\ -\,247 \\ \hline 631 \end{array}$$

6) $597 - 53 \longrightarrow$ $$\begin{array}{r} 597 \\ -\ \ 53 \\ \hline 544 \end{array}$$

Subtraction with borrowing

7) $$\begin{array}{r} \overset{9}{\cancel{10}}{}^{1}57 \\ -\ \ 2\,64 \\ \hline 793 \end{array}$$

8) $51187 - 1249 \longrightarrow$ $$\begin{array}{r} \overset{4}{\cancel{5}}\ \overset{10}{\cancel{1}}\ {}^{1}1\ \overset{7}{\cancel{8}}\ {}^{1}7 \\ -\quad 1\ 2\ 4\ 9 \\ \hline 4\ \ 9\ 9\ 3\ 8 \end{array}$$

**** Notice that we can add any number of numbers together and in any order. However, the order matters in subtraction and you can only subtract two values at a time.****

Multiplication:

9) $$\begin{array}{r} 4\,2\,3 \\ \times\quad 2 \\ \hline 8\,4\,6 \end{array}$$

10) $$\begin{array}{r} {\scriptstyle 5\ 5}\quad \\ 2\,6\,7 \\ \times\quad 8 \\ \hline 2\,1\,3\,6 \end{array}$$

11) $$\begin{array}{r} {\scriptstyle 5}\qquad \\ 3\,6\,1 \\ \times\quad 9 \\ \hline 3\,2\,4\,9 \end{array}$$

12)
$$\begin{array}{r} {\color{red}1\,2} \\ 523 \\ \times\ 17 \\ \hline 3661 \\ +\ 5230 \\ \hline 8891 \end{array} \quad \leftarrow \text{place holder}$$

13)
$$\begin{array}{r} {\color{red}2} \\ 47 \\ \times\ 23 \\ \hline 141 \\ +\ 940 \\ \hline 1081 \end{array} \quad \leftarrow \text{place holder}$$

14)
$$\begin{array}{r} {\color{red}3} \\ 6016 \\ \text{x}\ \ 146 \\ \hline 36096 \\ 240640 \\ +\ 601600 \\ \hline 878336 \end{array} \quad \leftarrow \text{place holder}$$

Long Division:

15)
$$\begin{array}{r} 329 \\ 7\overline{)\,2303} \\ -21 \\ \hline 20 \\ -14 \\ \hline 63 \\ -63 \\ \hline 0 \end{array}$$

16)
$$\begin{array}{r} 219 \\ 32\overline{)\,7019} \\ -64 \\ \hline 61 \\ -32 \\ \hline 299 \\ -288 \\ \hline 11 \end{array}$$

Quotient: $219\frac{11}{32}$

17)
$$\begin{array}{r} 2013 \\ 24\overline{)\,48327} \\ -48 \\ \hline 3 \\ -0 \\ \hline 32 \\ -24 \\ \hline 87 \\ -72 \\ \hline 15 \end{array}$$

Quotient: $2013\frac{15}{24}$

Signed Numbers:

6 --> positive "6" -6 --> negative "6"

Think about addition and subtraction of signed numbers like spending money.

17 – 9 = 8   "I have $17 and spend $9, I am left with $8"

9 – 17 = -8 " I have $9 and spend $17, I am short by $8, so I have negative $8"

$-11 - 27 = -38$ "I spend \$11 and spend another \$27, I have spent \$38, so I have negative \$38"

When multiplying and dividing signed numbers, there are two rules to keep in mind:

- if the signs are the same, either positive or negative --> the answer is *positive*.
- if the signs differ --> the answer is *negative*.

Example:

a. $6\,(-4) = -24$

b. $-12\,(12) = -144$

c. $\frac{36}{3} = 12$

d. $\frac{51}{-3} = -17$

*e. $\frac{0}{9} = 0$

f. $-7\,(-11) = 77$

g. $2\,(3)(-6) = -36$

h. $\frac{-25}{5} = -5$

i. $\frac{-121}{-11} = 11$

*j. $\frac{9}{0} =$ undefined

*** Special Cases: you cannot ever divide by "0".**

$\frac{0}{9} = 0$ and $\frac{9}{0} =$ undefined

Applications:

1) 78 + 23

2) 1623 + 287

3) 706 + 901

4) 197 + 13 + 682

5) 673 + 23 + 489

6) 11 + 907 + 556

7) 3261 + 573 + 10483

8) 105 + 2401 + 45106

9) 462 + 198

10) $673 - 28$

11) $9247 - 7348$

12) $37\ x\ 2$

13) $8074 - 2347$

14) $823\ x\ 6$

15) $6852 - 1748$

16) $1348\ x\ 5$

17) $9438 - 2789$

18) $45\ x\ 23$

19) $567 - 189$

20) $746\ x\ 62$

21) $6066 - 1879$

22) $812\ x\ 37$

23) $245\ x\ 847$

24) $9724 \div 27$

25) $2344\ x\ 6005$

26) $3817 \div 29$

27) $157 \div 24$

28) $73 - 81$

29) $53 \div 12$

30) $-41 + 8$

31) $7616 \div 7$

32) $2 - 67$

33) $8904 \div 42$

34) $-121 - 34$

35) $0 - 5$

36) $\frac{42}{3}$

37) $-8 - 14$

38) $\frac{-54}{9}$

39) $5\,(4)$

40) $\frac{-242}{-2}$

41) $-6\,(15)$

42) $\frac{0}{2}$

43) $-4\,(-21)$

44) $\frac{14}{0}$

45) $21\,(-7)$

46) $\frac{16}{0}$

47) $-16\,(-4)$

48) $\frac{0}{201}$

49) $-27\,(-31)$

50) $\frac{16}{-2}$

Topic 2 - Order of Operations

Vocabulary

Base - - > a number or variable that is raised to an exponent (power).

Exponent - - > a number that indicates the number of times the base is multiplied to itself.

Notation

exponent

5^3 $\qquad\qquad$ $5^3 = 5 \bullet 5 \bullet 5 = 125$

base

( ) , [ ], and { } are grouping symbols

Note the power of parenthesis:

$(-2)^3 = -2 \bullet -2 \bullet -2 = -8$, $\qquad -2^3 = -1 \bullet 2 \bullet 2 \bullet 2 = -8$

$(-3)^4 = -3 \bullet -3 \bullet -3 \bullet -3 = 81$, $\qquad -3^4 = -1 \bullet 3 \bullet 3 \bullet 3 \bullet 3 = -81$

Order of Operations

		$\longmapsto$		$\longmapsto$	
P	E	M	D	A	S
( )	exponents	*	÷	+	−

P -> parenthesis -> perform the operations within the parenthesis first
E -> exponents -> perform the exponents second
M-> multiplication } performed from left to right
D-> division
A ->addition } performed from left to right
S -> subtraction

* *quick way to remember the order is :* ***"Please Excuse My Dear Aunt Sally"***

Example:

1) $5 + (7 - 8 \div 4) + 3 = 5 + (7 - 2) + 3 = 5 + 5 + 3 = 13$
2) $-2^2 - 3\,(15 \div 5) - 8 = -4 - 3(3) - 8 = -4 - 9 - 8 = -21$
3) $2 + 10^2 \div 5 \bullet 2^2 = 2 + 100 \div 5 \bullet 4 = 2 + 20 \bullet 4 = 2 + 80 = 82$

4) $\frac{4 \div 2 \bullet 4^2 - 3 \bullet 2}{(7-4)^3 - 2 \bullet 5 - 4} = \frac{4 \div 2 \bullet 16 - 3 \bullet 2}{3^3 - 2 \bullet 5 - 4} = \frac{2 \bullet 16 - 3 \bullet 2}{27 - 2 \bullet 5 - 4} = \frac{32 - 6}{27 - 10 - 4} = \frac{26}{13} = 2$

5) $\frac{(3-5)^2 - (7-13)}{(2-5)3 + 2 \bullet 4} = \frac{(-2)^2 - (-6)}{(-3)3 + 2 \bullet 4} = \frac{4+6}{-9+8} = \frac{10}{-1} = -10$

Applications

Solve the following problems using the order of operations -> P E M D A S

1) $6 + 3 + 9$

2) $7 - 15 + 5$

3) $3 - (11 - 8)$

4) $4^2 + 3 \bullet 2 - 1^2$

5) $5 - (3^2 - 5)$

6) $2(8 - [6 - 2] + 3)$

7) $5 + 12(8) + 30 \div 6 + 8 - 5$

8) $16 \div 8(4 \div 2 + 2) + 7$

9) $7(6+3) - 18$

10) $-2 \bullet 5 + 12 \div 3$

11) $-4(9-8) + (-7)(2^3)$

12) $(8+6) \div 7 \bullet 3 - 6$

13) $(-4-1)(-3-5) - 2^3$

14) $5 - 4(5^2 - 9 \div 3)$

15) $4^2 + 12 \div 3 \bullet 2$

16) $(6-9)(-2-7) \div (-4)$

17) $7 + [6^2 \div 4(5-3)] + 11$

18) $3(18 - [2^2 + 7] - 2)$

19) $2^3 - 7^2 \div (2^2 - 5) \bullet 9$

20) $\frac{-8-4(-6)\div 12}{4+3}$

21) $\frac{15 \div 5 \bullet 4 \div 6 - 8}{-6+5-8 \div 2}$

22) $\frac{3^3 - 5}{2(-8) - 5(3)}$

23) $\frac{6(-4) - 3^2(2)^3}{-5(-2+6)}$

24) $\frac{(-7)(-3) + 2^3(-5)}{(-2^2 - 2)(-1-6)}$

Topic 3: Fraction Review

Vocabulary

denominator -> **bottom of fraction**
numerator -> **top of fraction**
improper fraction -> **fraction whose numerator is greater than the denominator**
proper fraction -> **fraction whose numerator is smaller than the denominator**

* We will give all solutions in the form of improper fractions, if necessary.

Notation: $\frac{5}{6}$ not 5/6

We need to make the fraction bar horizontal not diagonal.

To multiply fractions : "tops times tops and bottoms times bottoms"
To divide fractions : "stay, change, flip, and then multiply"
To add/subtract fractions : denominators must be the same, then apply the operation to the numerators only.

Examples:

Proper fraction : $\frac{2}{5}$, $\frac{11}{12}$, $\frac{7}{13}$

Improper fraction: $\frac{15}{12}$, $\frac{20}{7}$, $\frac{9}{2}$, $\frac{18}{8}$ *

* improper fractions must be reduced, if possible

$$\frac{18}{8} = \frac{9}{4} \; ; \quad \frac{24}{15} = \frac{8}{5} \; ; \quad \frac{9}{45} = \frac{1}{5} \; ; \quad \frac{14}{42} = \frac{2}{6} = \frac{1}{3}$$

Example (**multiplying fractions**):

1) $\frac{5}{8} \bullet \frac{6}{3} = \frac{30}{24} = \frac{5}{4}$ or $\frac{5}{\underset{4}{\cancel{8}}} \bullet \frac{\overset{\overset{1}{\cancel{2}}}{\cancel{6}}}{\cancel{3}} = \frac{5}{4}$

in some cases you may be able to cross-cancel before doing the multiplication (you may only cross-cancel when multiplying fractions !)

2) $\frac{\overset{\overset{4}{\cancel{12}}}{\cancel{48}}}{\underset{\cancel{3}}{\cancel{15}}} \bullet \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{4}}} = 4$

3) $\frac{9}{\underset{4}{\cancel{20}}} \bullet \frac{\overset{3}{\cancel{15}}}{8} = \frac{27}{32}$

Example (**dividing fractions**):

1) $\frac{2}{3} \div \frac{5}{12}$

- "**stay**" the first fraction stays the same
- "**change**" the $\div$ sign becomes multiplication
- "**flip**" invert the second fraction
- "**multiply**" multiply fractions

$$\frac{2}{3} \bullet \frac{12}{5} = \frac{2}{\underset{1}{\cancel{3}}} \bullet \frac{\overset{4}{\cancel{12}}}{5} = \frac{8}{5}$$

2) $\frac{11}{15} \div \frac{7}{3} = \frac{11}{\underset{5}{\cancel{15}}} \bullet \frac{\overset{1}{\cancel{3}}}{7} = \frac{11}{35}$

3) $\frac{1}{2} \div \frac{7}{8} = \frac{1}{3} \bullet \frac{8}{7} = \frac{8}{21}$

Adding / subtracting fractions:

The quickest way to find the LCD (least common denominator – a number that all the denominators divide into evenly) is to multiply the denominators together.

$\frac{1}{3} + \frac{1}{5}$ the LCD = 15 - → (3 * 5)

Steps in adding / subtracting fractions:

1) Determine the LCD
2) Multiply the first fraction by the second denominator on top and bottom. (we are only multiplying by "1" in a special form)
3) Multiply making the denominators the same.
4) Perform the operation (addition or subtraction) with the numerators.

$$\frac{1}{3}\left(\frac{5}{5}\right) + \frac{1}{5}\left(\frac{3}{3}\right) = \frac{5}{15} + \frac{3}{15} = \frac{5+3}{15} = \frac{8}{15}$$

Examples:

1) $\frac{7}{4} + \frac{2}{3} = \frac{7}{4}\left(\frac{3}{3}\right) + \frac{2}{3}\left(\frac{4}{4}\right) = \frac{21}{12} + \frac{8}{12} = \frac{29}{12}$

2) $\frac{12}{5} - \frac{2}{3} = \frac{12}{5}\left(\frac{3}{3}\right) - \frac{2}{3}\left(\frac{5}{5}\right) = \frac{36}{15} - \frac{10}{15} = \frac{26}{15}$

3) $\frac{6}{11} - \frac{5}{2} = \frac{6}{11}\left(\frac{2}{2}\right) - \frac{5}{2}\left(\frac{11}{11}\right) = \frac{12}{22} - \frac{55}{22} = -\frac{43}{22}$

Applications

Give all solutions in reduced fraction form.

1) $\frac{3}{4} \bullet \frac{1}{7} =$

2) $\frac{4}{7} \bullet \frac{14}{3} =$

3) $\frac{2}{9} \bullet \frac{3}{8} =$

4) $\frac{6}{7} \bullet \frac{18}{35} =$

5) $\frac{11}{15} \bullet \frac{27}{44} =$

6) $\frac{12}{5} \bullet \frac{1}{4} =$

7) $\frac{15}{2} \bullet \frac{1}{3} =$

8) $\frac{18}{3} \bullet \frac{1}{9} =$

9) $\frac{6}{5} \bullet \frac{3}{8} =$

10) $\frac{1}{2} \bullet \frac{1}{2} =$

11) $\frac{6}{5} \div \frac{3}{8} =$

12) $\frac{1}{2} \div \frac{7}{8} =$

13) $\frac{2}{3} \div \frac{9}{5} =$

14) $\frac{14}{3} \div \frac{7}{8} =$

15) $\frac{5}{6} \div \frac{1}{3} =$

16) $\frac{12}{8} \div \frac{14}{18} =$

17) $\frac{6}{11} \div \frac{1}{45} =$

18) $\frac{12}{5} \div \frac{1}{4} =$

19) $\frac{1}{2} + \frac{2}{5} =$

20) $\frac{11}{3} + \frac{4}{5} =$

21) $\frac{7}{10}+\frac{7}{3}=$

22) $\frac{8}{9}+\frac{1}{12}=$

23) $\frac{5}{8}+\frac{1}{30}=$

24) $\frac{9}{2}+\frac{6}{8}=$

25) $\frac{14}{5}-\frac{1}{2}=$

26) $\frac{3}{5}-\frac{1}{3}=$

27) $\frac{7}{10}-\frac{2}{5}=$

28) $\frac{18}{7}-\frac{9}{8}=$

29) $\frac{4}{9}-\frac{3}{8}=$

30) $\frac{9}{15}-\frac{5}{12}=$

Topic 4: Improper Fractions & Prime Factorization

<u>Vocabulary</u> :

Mixed number - -> a whole number with a fraction

ex. $4\frac{2}{5}$ or $11\frac{1}{4}$

*All fraction answers must be either a proper or improper fraction – no mixed numbers *

Prime number - -> special numbers that are only divisible by 1 and itself.

ex. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, ……….
(there is an infinite number of prime numbers)

Prime factors - -> prime numbers that multiply together to produce a number. Every number has a unique prime factorization. No two numbers have the same (exact) prime numbers that make it up.

<u>Converting mixed numbers into improper fractions</u>

numerator

$5\frac{2}{3}$

whole number   denominator

1. Multiply the whole number by the denominator
2. Add the numerator to that product
3. Put that number as the new numerator

$$5\frac{2}{3} = \frac{17}{3}$$

mixed number   Improper fraction

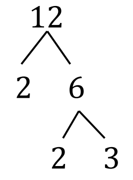
Ex. 1) $1\frac{7}{10} = \frac{17}{10}$ 2) $5\frac{3}{8} = \frac{43}{8}$ 3) $13\frac{1}{4} = \frac{53}{4}$

4) $8\frac{5}{7} = \frac{61}{7}$ 5) $21\frac{1}{2} = \frac{43}{2}$ 6) $13\frac{9}{10} = \frac{139}{10}$

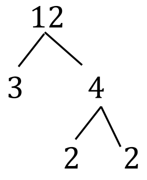
Finding prime factorization:

Using the "factoring tree"

Example:



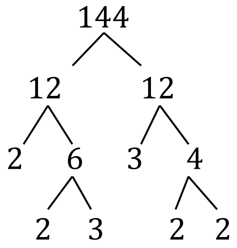
or

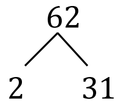


It does not matter how you begin breaking down the number. You will always get the same prime factorization.

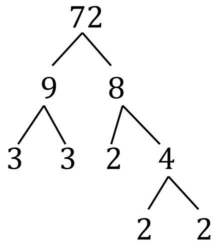
12 = 2•2•3

12 = 2•2•3



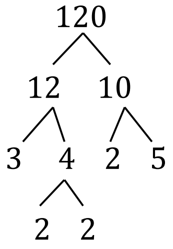


62 = 2•31

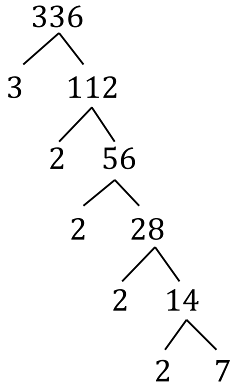


144 = 2•2•2•2•3•3

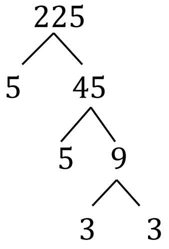
72 = 2•2•2•3•3



120 = 2•2•2•3•5



336 = 2•2•2•2•3•7



225 = 3•3•5•5

Applications

Rewrite the following mixed numbers in improper fraction form:

1) $3\frac{4}{7}$

2) $2\frac{1}{3}$

3) $7\frac{2}{5}$

4) $6\frac{3}{4}$

5) $5\frac{7}{8}$

6) $4\frac{1}{9}$

7) $1\frac{11}{13}$

8) $8\frac{3}{5}$

9) $9\frac{2}{3}$

10) $15\frac{4}{7}$

Find the prime factorization of the following numbers:

11) 28

12) 72

13) 56

14) 42

15) 108

16) 225

17) 63

18) 92

19) 144

20) 303

21) 525

22) 128

23) 87

24) 124

Topic 5 : Simplify Variable Expressions

Vocabulary:

Variable --> a number that is represented by a letter

Constant --> a number that stands alone

Coefficient --> a number attached to a variable

Term --> specific elements of an expression

Expression --> a string of constants, variable and/or variables with coefficient…with no equal signs.

Example:

$$x \quad - \quad y \quad + \quad 4m \quad + \quad 9np \quad + \quad 7 \qquad (total\ of\ 5\ terms)$$

x	y	$4m$	$9np$	7
variable	variable with coefficient of -1	variable with coefficient of 4	variable with coefficient of 9	constant

We may only combine terms with the exact same variables (we say….collect like terms)

Example:

1) $5x + 7y - 2x + 12y = 3x + 19y$

2) $6m - 10y + 4xy + 21y = 6m + 11y + 4xy$

3) $\frac{2}{5}a + \frac{1}{2} - \frac{1}{7}a - \frac{10}{3} = \frac{9}{35}a - \frac{17}{6}$

4) $5\,(2x - 3y) + 17x - 10$

distribute (or multiply) the "5" to all terms in the parenthesis

$10x - 15y + 17x - 10$ <-- now collect like terms

$27x - 15y - 10$

5) $3a(4 - 2b) + 15a + 17ab$

$= 12a - 6ab + 15a + 17ab$

$= 27a + 11ab$

6) $-2\,(7x + 4) - 3x - 1$

$= -14x - 8 - 3x - 1 = -17x - 9$

7) $6-(5x-1)+3x-12$
$=6-5x+1+3x-12$
$=-2x-5$

Applications

1) $2x+15y-7y$

2) $8a-4c+10c-3a$

3) $6w-3v-11w+4v-19$

4) $3-2x-6m+13-7x$

5) $4(x+y)-12y$

6) $9p+2(2p-7n)$

7) $3(5w+2-3y)+12$

8) $-5(12-8u)+72+6u$

9) $9y-2x(y+2)+9xy$

10) $2h-(5k-7h+2)+10k$

11) $23-(8+4x)+4x$

12) $4(2u-6v)-3(7v-11u)$

13) $9ab+18b-5a+2a(b+7)$

14) $-7(13+7x-12y)-x(2-y)$

15) $5-(w-3)+6(5+w)-21w$

16) $8ab(c-d)+4abd-9abc$

17) $\frac{4}{3}x-\frac{1}{5}x-4y+\frac{5}{3}y$

18) $\frac{7}{2}\left(\frac{1}{9}a-\frac{3}{4}\right)-\frac{2}{5}$

19) $\frac{9}{4}-\left(\frac{7}{8}k+\frac{6}{7}\right)-\frac{1}{12}$

20) $\frac{8}{5}f+\frac{1}{4}\left(\frac{2}{3}-6f\right)+\frac{2}{3}+\frac{5}{8}f$

Topic 6: Evaluating Variable Expressions

Evaluating variable expressions will use the topics we studied in the last 2 classes, simplifying variable expressions and using the order of operations.

Examples:

1) Evaluate $7x - 3$ where $x = 4$ (we replace the "x" with "4")

$$7(4) - 3 = 28 - 3 = 25$$

2) Evaluate $2(x + 5) - 6$ where $x = 4$

$$2(4 + 5) - 6 = 2(9) - 6 = 18 - 6 = 12$$

3) Evaluate $2(x - 5) - 6$ where $x = -3$

$$2(-3 - 5) - 6 = 2(-8) - 6 = -16 - 6 = -22$$

4) Evaluate $x^2 - 3x + 1$ where $x = 2$

$$(2)^2 - 3(2) + 1 = 4 - 6 + 1 = -1$$

5) Evaluate $x^2 - 3x + 1$ where $x = -4$

$$(-4)^2 - 3(-4) + 1 = 16 + 12 + 1 = 29$$

6) Evaluate $4xy + 5y - 2x$ where $x = 2$ and $y = -3$

$$4(2)(-3) + 5(-3) - 2(2) = -24 - 15 - 4 = -43$$

7) Evaluate $3(ab + 2b) + 4c$ where $a = -1$, $b = 2$, and $c = -2$

$$3[(-1)(2) + 2(2)] + 4(-2) = 3(-2 + 4) - 8 = 3(2) - 8 = 6 - 8 = -2$$

8) Evaluate $-2w^2 - 3wv + 5$ where $w = -3$ and $v = 5$

$$-2(-3)^2 - 3(-3)(5) + 5 = -2(9) + 45 + 5 = -18 + 50 = 32$$

*when you replace a variable with a number, it is important to use parenthesis when "plugging" the given values into the expression to preserve the signs and operations.

9) Evaluate $mn + 6(m - n) + 10$ where $m = 7$ and $n = -3$

$$(7)(-3) + 6(7 - (-3)) + 10 = -21 + 6(7 + 3) + 10 = -21 + 60 + 10 = 49$$

10) Evaluate $\frac{q^2 + 4qp}{(p-q) + 2}$ where $q = 3$ and $p = -2$

$$\frac{(3)^2 + 4(3)(-2)}{(-2 - 3) + 2} = \frac{9+12(-2)}{(-5)+2} = \frac{9-24}{-3} = \frac{-15}{-3} = 5$$

Applications

Evaluate the following expressions with the given substitutions:

1) $5(x - 3) + 6$ where a) $x = 5$

b) $x = -4$

2) $ab + 2(a + b)$ where a) $a = 3$ and $b = 2$

b) $a = -1$ and $b = -5$

3) $2(x + 5y) - xy + 3y$ where a) $x = 7$ and $y = 1$

b) $x = -3$ and $y = 2$

4) $\frac{mnp-(m-n)^2}{p^2+5n}$ where a) $m = 5$, $n = 2$, and $p = 4$

b) $m = -1$, $n = -3$, and $p = 3$

Topic 7: Solving Linear Equations

The key to solving linear equations is recognizing the operations occurring and remembering that when you move anything from one side of the equal sign to the other side, you must do the opposite operation.

operation	**opposite operation**
addition	**subtraction**
subtraction	**addition**
multiplication	**division**
division	**multiplication**

Steps to solve linear equations:

1) Remove parenthesis by distribution.
2) Collect like terms on each side of the equal sign.
3) Move all variables to left-side of the equation.
4) Move all constants to the right-side of the equation.
5) Divide by the number in front of the variable.

Examples:

1)

$$3x - 7 = 2$$
$$\underline{\quad +7 \quad +7 \quad}$$
$$\frac{3x}{3} = \frac{9}{3}$$
$$x = 3$$

2)

$$-4x + 3 = 6$$
$$\underline{\quad -3 \quad -3 \quad}$$
$$\frac{-4x}{-4} = \frac{3}{-4}$$
$$x = \frac{3}{-4}$$

3)

$$5x + 7 - 2x = x - 9$$
$$3x + 7 = x - 9$$
$$\underline{-x \qquad -x}$$
$$2x + 7 = -9$$
$$\underline{\quad -7 \quad -7 \quad}$$
$$\frac{2x}{2} = \frac{-16}{2}$$
$$x = -8$$

4)

$$6(x + 1) = 22$$
$$6x + 6 = 22$$
$$\underline{\quad -6 \quad -6 \quad}$$
$$\frac{6x}{6} = \frac{16}{6}$$
$$x = \frac{8}{3}$$

5)

$$-2(x - 3) + 4x = 3(x + 2)$$
$$\underline{-2x + 6 + 4x = 3x + 6}$$
$$2x + 6 = 3x + 6$$
$$-x + 6 = 6$$
$$x = 0$$

6)

$$\frac{1}{4}x + 5 = 11$$
$$\underline{\quad -5 \quad -5 \quad}$$
$$\frac{\frac{1}{4}x}{\frac{1}{4}} = \frac{6}{\frac{1}{4}}$$
$$x = 24$$

Applications

1) $5x + 2 = 47$

2) $2x + 3 = 15$

3) $2(3x - 1) = 10$

4) $3(x + 4) = 5x - 12$

5) $2x - 7 = 9$

6) $k + 27 = 12$

7) $-5p = -65$

8) $2m - 8m = 24$

9) $9x + 6 = 6x + 8$

10) $3(8x - 1) = 6(5 + 4x)$

11) $12y - 3(2y - 1) = 12$

12) $-2(y - 4) - (3y - 2) = -2$

13) $7(x + 1) = 4[x - (3 - x)]$

14) $5(x - 4) - 2x = x + 2(2 - x)$

15) $3 - (2x + 7) = 3x + 1 - (5x + 8)$

16) $5[2 - (2x - 4)] = 3(3 - 3x)$

17) $\frac{2}{5}x + 6 = 4$

18) $4 - \frac{3}{8}a = -2$

19) $20 - \frac{x}{3} = \frac{x}{2}$

20) $\frac{2x}{3} = \frac{x}{6} + 1$

21) $\frac{2}{3}n - \frac{1}{6} = -\frac{1}{3}$

22) $\frac{x}{2} - \frac{1}{10} = \frac{x}{5} + \frac{1}{2}$

23) $\frac{5}{2} - \frac{x-3}{3} = 2$

24) $5[2 - (2x - 4)] = 2(5 - 3x)$

Topic 8: Exponent Rules

Rules:

1) $X^m \bullet X^n = X^{m+n}$
2) $(XY)^n = X^nY^n$
3) $\frac{X^m}{X^n} = X^{m-n}$
4) $(\frac{X}{Y})^n = \frac{X^n}{Y^n}$
5) $(X^m)^n = X^{mn}$

Examples: Simplify the following exponential expressions

1) $(4a^2 b^3)(-3a^4b) = -12a^6b^4$

2) $(-5a^7c^9)(-2ac^{12}) = 10a^8c^{21}$

3) $\frac{15x^9y^6}{5x^6y^2} = 3x^3y^4$

4) $\frac{56m^5n^3}{8m^2n^3} = 7m^3$

5) $\frac{27p^3q^5}{12p^2q} = \frac{9pq^4}{4}$

6) $(2k^4)^2 = (2)^2(k^4)^2 = 4k^8$

7) $(-3m^2)^3 = (-3)^3(m^2)^3 = -27m^6$

8) $(\frac{4}{x^3y})^3 = \frac{(4)^3}{(x^3)^3(y)^3} = \frac{64}{x^9y^3}$

9) $(\frac{2a^2c^3}{4a^5c^2})^2 = (\frac{c}{2a^3})^2 = \frac{(c)^2}{(2)^2(a^3)^2} = \frac{c^2}{4a^6}$

simplify inside first

10) $(\frac{15w^8y^3}{5w^6y^9})^3 = (\frac{3w^2}{y^6})^3 = \frac{(3)^3(w^2)^3}{(y^6)^3} = \frac{27w^6}{y^{18}}$

Applications:

Simplify the following exponential expressions completely; be sure that there are no negative exponents in the solutions:

1) $(2xy)(-3x^2yz)(x^3y^3z^3)$

2) $(5x^2y^4z)(-3xyz)$

3) $(2a^3b^2c)(-11a^3bc^2)$

4) $\frac{49a^3bc^{14}}{-7abc^{16}}$

5) $(2d^5)^3$

6) $(3x^4)^3$

7) $(5^2x^4)^2(2^2x^6)^3$

8) $(\frac{m^3n^2}{p^2})^4$

9) $(3x^4y^{-4}z^{-3})^2$

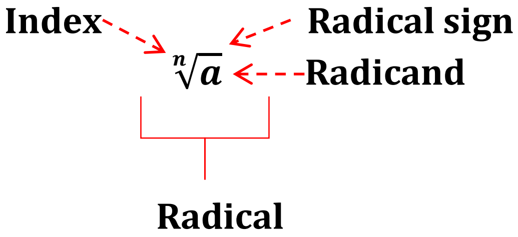
10) $(\frac{-18m^2n^{-3}p}{3m^5n^2p^{-2}})^3$

11) $(\frac{-24x^{-3}y^4}{6x^5y^{-7}z})^2$

12) $\frac{(3x^{-2}y)^2}{(4xy^{-1})^3}$

Topic 9: Radicals

The number a is the **radicand.**
n is the **index** or **order.**
The expression $\sqrt[n]{a}$ is the **radical**.



Example 1

Simplify:

a. $\sqrt[3]{27} = 3$, because $3^3 = 27$

b. $\sqrt[3]{216} = 6$, because $6^3 = 216$

c. $\sqrt[4]{256} = 4$, because $4^4 = 256$

d. $\sqrt[5]{243} = 3$, because $3^5 = 243$

e. $\sqrt[4]{\frac{16}{81}} = \frac{2}{3}$, because $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$

f. $\sqrt[3]{0.064} = 0.4$, because $0.4^3 = 0.064$

Example 2

Find each root:

a. $\sqrt{36} = 6$

b. $-\sqrt{36} = -6$

c. $\sqrt[4]{16} = 2$

d. $-\sqrt[4]{16} = -2$

e. $\sqrt[4]{-16}$ *(not a real number)*

f. $\sqrt[5]{243} = 3$

g. $\sqrt[5]{-243} = -3$

Example 3

Simplify. Assume that all variables represent positive real numbers.

a. $\sqrt{25p^7} = \sqrt{5^2 \bullet (p^3)^2 \bullet p} = 5p^3\sqrt{p}$

b. $\sqrt{72y^3x} = \sqrt{36 \bullet 2 \bullet y^2 \bullet y \bullet x} = 6y\sqrt{2yx}$

c. $\sqrt[3]{-27y^7x^5z^6} = \sqrt[3]{-3^3 \bullet y^6 \bullet y \bullet x^3 \bullet x^2 \bullet z^6} = -3y^2xz^2\sqrt[3]{yx^2}$

d. $-\sqrt[4]{32a^5b^7} = -\sqrt[4]{2^4 \bullet 2 \bullet a^4 \bullet a \bullet b^4 \bullet b^3} = -2ab\sqrt[4]{2ab^3}$

Applications

1) $\sqrt{48}$

2) $\sqrt[3]{135}$

3) $\sqrt[3]{m^6n^4}$

4) $\sqrt{x^9y^{11}}$

5) $\sqrt{81x^{15}y^{21}}$

6) $\sqrt{484x^5yw^{10}}$

7) $\sqrt[3]{192a^5b^{12}c^8}$

8) $\sqrt[3]{500k^6m^{14}}$

9) $\sqrt[4]{144p^9w^{13}}$

10) $\sqrt[5]{974x^{13}y^4}$

11) $\sqrt{84t^8n^{11}}$

12) $\sqrt[3]{625x^{28}y^{56}z^{112}}$

Topic 10: Addition & Subtraction of Polynomials

* Remember you can only combine like terms, the variables must match exactly*

Examples (addition):

1) $(4x - 5y + 2) + (7x - y - 10)$

You can rewrite the terms vertically lining each variable up under its match.

$$\begin{array}{rrr} 4x & -5y & +2 \\ 7x & -y & -10 \\ \hline 11x & -6y & -8 \end{array}$$

2) $(2x^2 + 7x + 6) + (-3x^2 - 11x - 10)$ $\dashrightarrow$

$$\begin{array}{rrr} 2x^2 & +7x & +6 \\ -3x^2 & -11x & -10 \\ \hline -x^2 & -4x & -4 \end{array}$$

3) $(9x^2y^2 + 5x^2y - 3y + 2) + (-6x^2y^2 - 3xy^2 - 15)$ $\dashrightarrow$

$$\begin{array}{rrrrr} 9x^2y^2 & +5x^2y & -3y & +2 & \\ -6x^2y^2 & & & -15 & -3xy^2 \\ \hline 3x^2y^2 & +5x^2y & -3y & -13 & -3xy^2 \end{array}$$

Examples (subtraction):

*Subtraction is done the same way, however, because we will be subtracting it is necessary to *<u>change all the signs in the second set of parenthesis</u>**

1) $(5m + 2n - 9) - (14m + 8n - 12)$

or $(5m + 2n - 9) + (-14m - 8n + 12)$

$$\begin{array}{rrr} 5m & +2n & -9 \\ -14m & -8n & +12 \\ \hline -9m & -6n & +3 \end{array}$$

2) $(3x^3 - 8x^2 + 5x + 2) - (-5x^3 + 21x - 6)$

or $(3x^3 - 8x^2 + 5x + 2) + (5x^3 - 21x + 6)$

$$\begin{array}{rrrr} 3x^3 & -8x^2 & +5x & +2 \\ 5x^3 & & -21x & +6 \\ \hline 8x^3 & -8x^2 & -16x & +8 \end{array}$$

3) $(2x^2y + 8xy - 4x - 1) - (7x^2y + 10xy + 6y + 5)$

or $(2x^2y + 8xy - 4x - 1) + (-7x^2y - 10xy - 6y - 5)$

$$\begin{array}{rrrrr} 2x^2y & +8xy & -4x & -1 & \\ -7x^2y & -10xy & & -5 & -6y \\ \hline -5x^2y & -2xy & -4x & -6 & -6y \end{array}$$

Applications

1) $(11x + 8y + 1) + (-8x - 13y - 2)$

2) $(11d - 4c + 6) - (17d + 10c + 11)$

3) $(3m^2 - 8m - 2) + (-16m^2 + 9m + 12)$

4) $(3x^2 + 2x - 6) + (13x^2 - 16x + 15)$

5) $(7x^2y + xy + 4) - (9x^2y - xy + 9x + 8)$

Topic 11: Multiplication of Polynomials

<u>Vocabulary</u> :

Monomial --> a single term variable expression
Binomial --> two variable expressions that are connected by addition or subtraction
Trinomial --> three variable expressions that are connected by addition or subtraction
Polynomial --> a series of many variable expressions that are connected by addition or subtraction

Examples:

monomial --> "$2x$" or "3" or "$7xy$"
binomial --> $(x + y)$ or $(2 + y)$
trinomial --> $(x + y - 2)$ or $(x^2 + x + 1)$
polynomial --> $(x^2 + 6x + 9)$ or $(x^3 - 7x^2 + x - 5)$

"polynomial" is the most commonly used term and applies to binomials and trinomials most often.

Examples:

1) $2(x + y) = 2x + 2y$
2) $2x(x + y) = 2x^2 + 2xy$
when you multiply like variables together you add the exponents together
3) $2x^2(x - 5) = 2x^3 - 10x^2$
4) $3xy(2x + 3y) = 6x^2y + 9xy^2$
5) $ab(3a - 2b + 4) = 3a^2b - 2ab^2 + 4ab$

<u>Now to binomials</u>

6) $(2x + 1)(x - 3)$
$2x(x) + 2x(-3) + 1(x) + 1(-3)$
$2x^2 - 6x + 1x - 3$
$= 2x^2 - 5x - 3$

Each term in the first set of parenthesis must be multiplied to each term in the second, then collect any like

7) $(4 + 2y)(y + 3)$
$4(y) + 4(3) + 2y(y) + 2y(3)$
$4y + 12 + 2y^2 + 6y$
$= 2y^2 + 10y + 12$

8) $(3a - 4c)(2a - 5c)$
$3a(2a) + 3a(-5c) + (-4c)(2a) + (-4c)(-5c)$
$6a^2 - 15ac - 8ac + 20c^2$
$= 6a^2 - 23ac + 20c^2$

***when you have two variables, write them in alphabetical order; so xy and yx are the same and db = bd ***

9) $(2m - 3)(2m + 5)$
$2m(2m) + 2m(5) + (-3)(2m) + (-3)(5)$
$4m^2 + 10m - 6m - 15$
$= 4m^2 + 4m - 15$

10) $(x + 4)(3x^2 - 2x + 5)$
$x(3x^2) + x(-2x) + x(5) + 4(3x^2) + 4(-2x) + 4(5)$
$3x^3 - 2x^2 + 5x + 12x^2 - 8x + 20$
$= 3x^3 + 10x^2 - 3x + 20$

11) $(x^2 + 5x + 7)(3x - 1)$
$x^2(3x) + x^2(-1) + 5x(3x) + 5x(-1) + 7(3x) + 7(-1)$
$3x^3 - x^2 + 15x^2 - 5x + 21x - 7$
$= 3x^3 + 14x^2 + 16x - 7$

12) $(2m^2 - 3m + 2)(m - 2)$
$2m^2(m) + 2m^2(-2) - 3m(m) - 3m(-2) + 2(m) + 2(-2)$
$2m^3 - 4m^2 - 3m^2 + 6m + 2m - 4$
$= 2m^3 - 7m^2 + 8m - 4$

13) $(5p^2 + 2p + 1)(2p^2 - 7p - 2)$
$= 10p^4 - 35p^3 - 10p^2 + 4p^3 - 14p^2 - 4p + 2p^2 - 7p - 2$
$= 10p^4 - 31p^3 - 22p^2 - 11p - 2$

Applications

1) $x(x + 6)$

2) $3x(x - 4)$

3) $7y(3y + 5)$

4) $2ac(a + 9c)$

5) $-2mn(5m - n)$

6) $(x + 2)(x + 3)$

7) $(y - 9)(y + 1)$

8) $(n - 4)(n - 5)$

9) $(3y + 1)(4y - 1)$

10) $(5p - 2)(5p - 2)$

11) $(2x - 5)(2x + 5)$

12) $(3ac - d)(ac + 6d)$

13) $(x - 5xy)(3x - 2xy)$

14) $(a + b)(c + d)$

15) $(3x^2 - 2x)(4x + 5)$

16) $(y - 2)(y^2 - 2y - 2)$

17) $(3x + 7)(2x^2 + 4x - 1)$

18) $(9m + 1)(m^2 - 5)$

19) $(6xy + 3)(x^2y + 7y - 4)$

20) $(8m + 5)(m^2 + 2m + 3)$

21) $(x^2 - x - 1)(x - 1)$

22) $(2c^2 + 6c + 3)(6 - 3c)$

23) $(a^2 + a - 4)(3a + 5)$

24) $(y^2 + 5y + 7)(y^2 + 2y - 3)$

Topic 12: Factoring Out GCF

Vocabulary:

GCF -> Greatest Common Factor is the largest number/power of a variable that is Common in an expression.

Factor out the GCF -> dividing out the common GCF from each term of an expression.

Examples:

1) $3x + 18$ -------> the **3** is common
$3(x + 6)$

2) $5x^2 - 15x + 35$ --> the **5** is common
$5(x^2 - 3x + 7)$

3) $2x^3 + 6x^2 - 8x$ ---> the **2x** is common
$2x(x^2 + 3x - 4)$

4) $6x^4 - 10x^3$ -----> the **$2x^3$** is common
$2x^3(3x - 5)$

5) $17x^3y^3 - 34x^3y^2 + 51x^2y$ -----> the **$17x^2y$** is common
$17x^2y(xy^2 - 2xy + 3)$

if the first term is negative, the negative sign is considered common and must be factored out as well

6) $-6m^3 - 12m^2 + 3m$ -----> the **$-3m$** is common
$-3m(2m^2 + 4m - 1)$ note that $\frac{3m}{-3m} = -1$

7) $-2x^5 - 2x^4 + 14x^3 + 2x$ ---> the **$-2x$** is common
$-2x(x^4 + x^3 - 7x^2 - 1)$

8) $-27x^2 + 9x + 6$ ---------> the **-3** is common
$-3(9x^2 - 3x - 2)$

Factoring can always be checked by multiplying the factors together

9) $8y^4 - 20y^3 + 12y^2 - 16y$ ---> the **4y** is common
$4y(2y^3 - 5y^2 + 3y - 4)$

now to check:

$$4y(2y^3 - 5y^2 + 3y - 4)$$
$$= 8y^4 - 20y^3 + 12y^2 - 16y$$

Applications

1) $3x + 36$

2) $7x^2 - 42x$

3) $-2x^3 - 6x^2 + 10x$

4) $-5x^3 + 35x - 25x$

5) $24y^3 - 18y^2 + 4y$

6) $-2x^5y^3 - 8x^4y^3 + 12x^4y^2 - 2x^4y^3$

Topic 13: Factoring Quadratic Expressions

*There are several factoring methods....use whichever method you prefer as long as you get the correct answer. This is one method that is very effective. It is called "AC method of factoring".

Vocabulary :

Descending order: an equation that reads from left to right, the highest power of x down to the constant.

Example: $5x^2 - 2x - 7 = 0$ <-------- equation in descending order & set equal to "0".

Remember that factoring can always be checked by multiplying the factors together to achieve the original equation back.

Steps for AC method :

1) Put equation in descending order.
2) Factor out any common number so that the factoring is more simple.
3) Multiply the leading coefficient and the constant.
4) Make a complete list of all pairs that multiply together to equal the product.
5) Choose from the list the pair that would sum to equal the coefficient of the middle variable.
6) Rewrite the middle variable using the chosen pair.
7) Split the now 4-piece expression in half and pull the common term out of the first half producing a "twin".
8) Write the "twin" down again and find the number necessary to multiply the twin by to match the second half of the expression.
9) The factors are: the "twin" and the expression surrounding the twin.

Example: $2x^2 - 7x + 6$

$2(6) = 12$

$\wedge$

1 12

-1 -12

2 6

-2 -6

3 4

$\boxed{-3 \quad -4}$

$2x^2 - 3x \mid - 4x + 6$

$x(2x - 3) - 2(2x - 3)$

$(2x - 3)(x - 2)$

$3x^2 + 14x + 15$

$3(15) = 45$

$1 \quad 45 \qquad \boxed{5 \quad 9}$

$-1 \quad -45 \qquad -5 \quad -9$

$3 \quad 15$

$-3 \quad -15$

$3x^2 + 5x \mid + 9x + 15$

$x(3x + 5) + 3(3x + 5)$

$(3x + 5)(x + 3)$

$2x^2 + 11x - 6$

$2(-6) = -12$

$1 \quad -12$

$\boxed{-1 \quad 12}$

$2 \quad -6$

$-2 \quad 6$

$3 \quad -4$

$-4 \quad 3$

$2x^2 - 1x \mid + 12x - 6$

$x(2x - 1) + 6(2x - 1)$

$(2x - 1)(x + 6)$

$4x^2 - x - 14$

$4(-14) = -56$

$1 \quad -56$

$-1 \quad 56$

$2 \quad -28$

$-2 \quad 28$

$4 \quad -14$

$-4 \quad 14$

$\boxed{7 \quad -8}$

$-7 \quad 8$

$4x^2 - 8x \mid + 7x - 14$

$4x(x - 2) + 7(x - 2)$

$(x - 2)(4x + 7)$

$12x^2 + 13x + 3$

$12(3) = 36$

1 36
-1 -36
2 18
-2 -18
3 12
-3 -12
$\boxed{4 \quad 9}$
-4 -9
6 6
-6 -6

$12x^2 + 4x \,|\, + 9x + 3$

$4x(3x + 1) + 3(3x + 1)$

$(3x + 1)(4x + 3)$

Applications

Factor completely.

1) $6x^2 - 7x - 10$

2) $2x^2 + 5x - 12$

3) $x^2 + x - 72$

4) $5x^2 - 14x - 3$

5) $8x^2 - 26x + 15$

6) $12x^2 - 17x + 6$

7) $6x^2 + 7x + 2$

8) $10x^2 + 27x + 5$

9) $6x^2 - 5x - 4$

10) $x^2 + 8x + 16$

11) $x^2 + x - 12$

12) $x^2 - 6x - 7$

13) $3x^2 + 16x + 16$

14) $x^2 - 19x + 40$

Topic 14: Special Factoring

Difference of Squares:

For equations that can be written in the form $(F)^2 - (L)^2$, there is a formula to use:

$$(F)^2 - (L)^2 = (F - L)(F + L)$$

Examples:

1) $x^2 - 16$
$(x)^2 - (4)^2$
$(x - 4)(x + 4)$

2) $16y^2 - 49$
$(4y)^2 - (7)^2$
$(4y - 7)(4y + 7)$

3) $100a^2 - 81b^2$
$(10a)^2 - (9b)^2$
$(10a - 9b)(10a + 9b)$

4) $4x^6 - y^4$
$(2x^3)^2 - (y^2)^2$
$(2x^3 - y^2)(2x^3 + y^2)$

5) $9m^8 - 1$
$(3m^4)^2 - (1)^2$
$(3m^4 - 1)(3m^4 + 1)$

6) $25x^2y^4 - w^6$
$(5xy^2)^2 - (w^3)^2$
$(5xy^2 - w^3)(5xy^2 + w^3)$

7) $36x^2 - 64y^{10}$
$(6x)^2 - (8y^5)^2$
$(6x - 8y^5)(6x + 8y^5)$

Difference/Sum of Cubes :

For equations that can be written in the form $(F)^3 \pm (L)^3$, there is a formula to use:

$$\left.\begin{aligned}(F)^3 + (L)^3 &= (F + L)(F^2 - FL + L^2)\\ (F)^3 - (L)^3 &= (F - L)(F^2 + FL + L^2)\end{aligned}\right\}$$

the quadratic factor is non-factorable always

Examples:

1) $8x^3 - 64$
$(2x)^3 - (4)^3$
$(2x - 4)(4x^2 + 8x + 16)$

2) $125x^3 + 27$
$(5x)^3 + (3)^3$
$(5x + 3)(25x^2 - 15x + 9)$

3) $216y^3 - 343$
$(6y)^3 - (7)^3$
$(6y - 7)(36y^2 + 42y + 49)$

4) $m^{12} + 8$
$(m^4)^3 + (2)^3$
$(m^4 + 2)(m^8 - 2m^4 + 4)$

5) $27n^9 - 125$
$(3n^3)^3 - (5)^3$
$(3n^3 - 5)(9n^6 + 15n^3 + 25)$

Applications

Factor completely.

1) $6x^2 + 34x + 20$

2) $100x^2 - 49$

3) $27m^3 - 64$

4) $3x^2 - 4x + 1$

5) $w^2v^2 - h^4k^4$

6) $64y^2 + 125$

7) $n^2 + n - 30$

8) $9a^6 - 4$

9) $y^9 - 8$

10) $2x^2 - 11x - 63$

11) $4p^2 + 15p + 9$

12) $81x^4 - 1$

13) $8x^{12} + 27$

14) $k^{18} - 125$

Topic 15: Solving Quadratic Equation Using Quadratic Formula

Quadratic Formula

The solutions of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1

Solve $4x^2 - 11x - 3 = 0.$

$$\downarrow \qquad \downarrow \qquad \searrow$$

a=4 b=-11 c=-3

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-11)\pm\sqrt{(-11)^2-4(4)(-3)}}{2(4)} = \frac{11 \pm\sqrt{121+48}}{8} = \frac{11 \pm\sqrt{169}}{8}$$

$$x = \frac{11 \pm 13}{8} \dashrightarrow x = \frac{11 + 13}{8} = \frac{24}{8} = 3$$

$$x = \frac{11 - 13}{8} = \frac{-2}{8} = -\frac{1}{4}$$

Therefore, the solution set is $\{-\frac{1}{4}, 3\}$.

Example 2

Solve $2x^2 + 19 = 14x.$

$$2x^2 - 14x + 19 = 0$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

a= 2 b= –14 c= 19

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-14)\pm\sqrt{(-14)^2-4(2)(19)}}{2(2)} = \frac{14 \pm\sqrt{196-152}}{4} = \frac{14 \pm\sqrt{44}}{4}$$

$$x = \frac{14\pm\sqrt{4\bullet 11}}{4} = \frac{14\pm 2\sqrt{11}}{4} \dashrightarrow x = \frac{14 + 2\sqrt{11}}{4} = \frac{2(7+\sqrt{11})}{4} = \frac{7+\sqrt{11}}{2}$$

$$x = \frac{14 - 2\sqrt{11}}{4} = \frac{2(7-\sqrt{11})}{4} = \frac{7-\sqrt{11}}{2}$$

Therefore, the solution set is $\{\frac{7 \pm \sqrt{11}}{2}\}$.

Example 3

Solve $(x+5)(x+1) = 10x$

$$x^2 + 6x + 5 = 10x$$

$$x^2 - 4x + 5 = 0$$

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a= 1 b= −4 c= 5

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)} = \frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm \sqrt{-4}}{2}$$

$$x = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm 2i}{2} \dashrightarrow x = \frac{4+2i}{2} = \frac{2(2+i)}{2} = 2+i$$

$$x = \frac{4-2i}{2} = \frac{2(2-i)}{2} = 2-i$$

Therefore, the solution set is $\{2 \pm i\}$.

Applications

Solve

1) $x^2 - 10x - 4 = 0$

2) $x^2 + 3x - 28 = 0$

3) $x^2 + 7x - 2 = 0$

4) $2x^2 - 3x - 1 = 0$

5) $2x^2 = 9x + 5$

6) $3x^2 - 4x = 3$

7) $x^2 - 22x + 102 = 0$

8) $x^2 = -10x + 4$

9) $6x^2 + 11x - 10 = 0$

10) $4x^2 + 12x = 7$

11) $2x^2 + 3x - 17 = 0$

12) $2x^2 = 5x + 12$